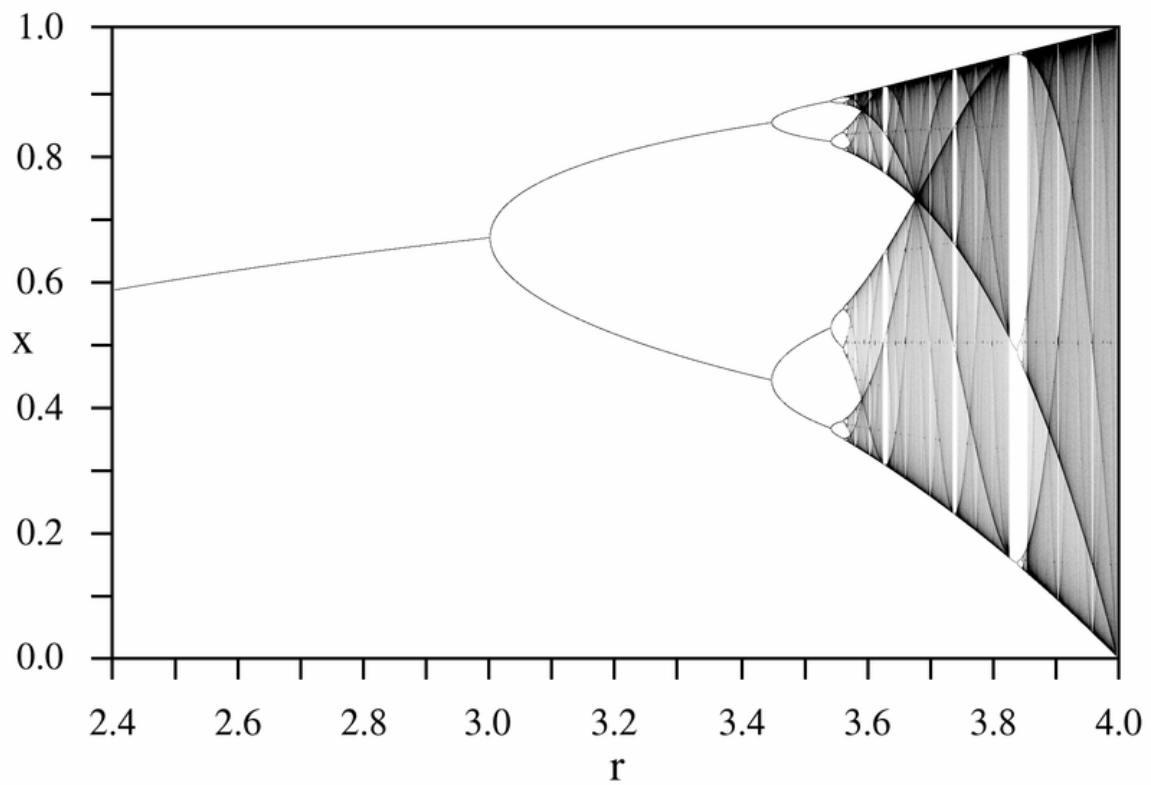


Templates for the Presentation of Simulation Results



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Foreword

Presentation of the results of simulation is a very important aspect of the study of a Dynamic System. It is indeed not rare to run simulations for dozens of combinations of parameters if the bifurcation diagram is a bit complex (by which I mean made of a few different regions). Most of the time the study of the system can be summed up with the detailed study of a few representative combinations of parameters (all regions of the bifurcation diagram must be represented of course). In which case the bulk of the simulations adds very little to the analysis and is only necessary to prove that nothing 'interesting' - noteworthy- happens between the cases that are studied in most details. These 'complementary' simulations are usually relegated to the appendix of the study report.

We detail in this document a simple template to present the basic simulation results of a Dynamic System. The template works best for 2D systems but it can easily be adapted to higher dimensions. The underlying principle of the template is that information on the parameters of the simulation (and on the corresponding theoretical predictions) should be appended next to the simulated Vector Field. Using the bifurcation diagram as a map is a powerful graphical way to achieve this.

We also detail in this document two templates for the analysis of some of the most important properties of a Vector Field. The first one deals with the analysis of a limit cycle. It can be easily adapted to the analysis of other similar objects (such as a basin of attraction). The second template deals with criteria that depend on the initial condition of a trajectory and sensitivity analysis.

Please Note: It is of course allowed – and recommended – to change the template in order to adapt to the various cases you encounter – for instance if your discussion is longer than the space allowed in the template or ,even better, if you have adopted a more sophisticated and effective way to visualize your results.

Analysis of the 'Name of Model' Model

Study Number

Description of Studied Point

Give :

- Parameters of the Model
- Description of the Theoretical results

For Instance:

A=1 B=1 C=0.1 D=5
Zone 2 of the Bifurcation Diagram
2 Steady Points:
(0,0) $\text{Det}(J) < 0$ $\text{Tr}(J) > 0$
Other Point : $\text{Det}(J) > 0$ $\text{Tr}(J) > 0$

Bifurcation Diagram

Acts as a map (where are we in the study?)

Show only relevant part of the diagram if needed.

Indicate studied point in Red

Vector Field for the Chosen Parameters

Must Show the Most Important Properties

- Steady Points
- Behaviour at Vicinity of Steady Points
- Limit Cycles
- A few well chosen trajectories
 - that converge to limit cycles
 - that converge to steady points
 - That diverge to infinity
 - etc...

If possible, should show the isoclines

Qualitative Description of the Vector Field

Insist on how simulations back theoretical results

Describe non-predicted results such as shapes, locations,...

Analysis of the Limit Cycle

Plots of $X(t)$, $Y(t)$... for the Limit Cycle

Qualitative Description of the Limit Cycle

Describe and Discuss such as shapes, locations,...

Quantitative Analysis of the Limit Cycle

Introduce your criteria if it is the first time you use them!!!

Discuss your results!

Sensitivity Analysis

Map for Criterion of Interest

Choose Criterion of Interest

Create Grid (use step that is small enough)

Compute criterion for every point of the grid by using point as initial point of a simulation

Example of criterion for Lotka Volterra

We know that for the model the trajectories are closed. The period of the orbit is a very interesting criterion for the study of the model.

Corresponding Sensitivity Map ('Heat' Map)

Define a sensitivity measure that relates to the way the criterion of interest changes in the vicinity of a point.

Compute the sensitivity criterion for every point of the grid

Example of criterion for Lotka Volterra

The criterion $C(X_0, Y_0)$ remains the period of the orbit. A possible sensitivity criterion is the maximum error committed by changing the initial conditions by 1% or less ie

$$S(X_0, Y_0) = \text{MAX} (|C(X, Y) - C(X_0, Y_0)|) \text{ for } |X - X_0| \leq 0.01 X_0 \text{ and } |Y - Y_0| \leq 0.01 Y_0$$

Analysis of the Sensitivity Map

Discuss your results!